Effects of the surroundig fluid on the dynamic characteristics of circular plates

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Objectives

Calculate the natural frequencies of a circular plate vibrating in a fluid incompressible or compressible

Existing Methods

Based on Energy conservation

Kinetic (plate+ fluid) + Potential

Aplication

incompressible
baffled plate

Lamb, McLachlan,Amabili

Fluid potential Fluid Kinetic energy

Gallego-Juárez

Added density plate due fluid acoustic pressure

Deformation modes

Polynomial

Vaccum modes

Present Method

BEM

Aplication

Incompressible compressible
Unbaffle plate

* Influence of the surrounding fluid on dynamic characteristics of structures known for many years

* Most works concerned underwater applications, sonar of a submarine

* The surrounding fluid was considered a liquid incompressibility

* Recently acoustical and spatial applications antennas or very light panels influence surrounded fluid of low density has been studied

* Avoided test structure vacuum chamber large antenna difficult expensive

* Spacecraft structures in payload compartment submitted intense vibration launcher lift- off

* Structures sandwich composite materials, low mass influence fluid low density important

* These structures, were modeled as baffled only recent research unbaffled plates.

* Two methods, (BEM) and (FEM).

* FEM at unbounded fluid domains:

- large size systems
- high computational cost
- Sommerfield radiation condition difficult to impose at the external mesh boundary.

* B. E. M. is an alternative method fluid is unbounded.

* Response of an unbaffled circular plate with arbitrary boundary conditions immersed in a fluid is calculated.

*A BEM compute the pressure jump over the plate

* Circular rings number of elements accurate result very low

* Kirchhoff’s integral formulation
  Helmholtz equation
  Sommerfield radiation condition

* Generalized forces
  Vacuum modes analitically
  Base functions displacement.

* Iteration
  natural frequencies
  compressible fluid.

Problem formulation

The deformation equation

$$D\nabla^4 w(r,\theta,t) + \rho_p h \frac{\partial^2 w}{\partial t^2} = \Delta p(r,\theta,t)$$

$$D = \frac{Eh^3}{12\left(1-\upsilon^2\right)}$$

Vibration frequency ω

$$w(r,\theta,t) = \tilde{w}(r,\theta)\cdot e^{-i\omega t}$$

$$p(r,\theta,z,t) = \tilde{p}(r,\theta,z)\cdot e^{-i\omega t}$$

Deformation equation spatial

$$D\nabla^4\tilde{w} - \rho_p h\omega^2\tilde{w} = \Delta\tilde{p}(r,\theta)$$

Fluid-Plate Boundary condition

$$\frac{\partial p}{\partial z} = -\rho_\infty \frac{\partial^2 w}{\partial t^2} \qquad z = 0$$

$$\frac{\partial \tilde{p}(r,\theta,z)}{\partial z} = \rho_\infty \omega^2 \tilde{w}(r,\theta)$$

Deformation plate Vacuum

$$\tilde{w}(r,\theta) = W_m^n(r)\cdot\cos(m\theta)$$

$$W_m^n\left(r\right) = A_m^n J_m(\beta_m^n r) + B_m^n I_m(\beta_m^n r)$$

m nodal diameters

n nodal circles

Bessel functions

Parameter β_m^n

Elastic dynamic characteristics

Vacuum frequencies

$$\omega_m^n = \beta_m^{n\,2}\sqrt{\frac{D}{\rho_p h}}$$

Sound Pressure equation

$$\frac{1}{a_\infty{}^2}\frac{\partial^2 p}{\partial t^2} - \Delta p = 0$$

$$\Delta\tilde{p} + k^2\,\tilde{p} = 0$$

Wave number

$$k = \frac{\omega}{a_\infty}$$

Kirchhoff-Helmholtz integral equation

$$\tilde{p}(x,y,z) = -\frac{1}{4\pi}\iint_{S_p} \Delta\tilde{p}(\xi,\eta)\frac{\partial}{\partial z}\left(\frac{e^{ikR}}{R}\right)d\xi d\eta$$

$$R = \left|\vec{x} - \vec{\xi}\right|$$

Green`s function $$g = -\frac{1}{4\pi}\frac{e^{ikR}}{R}$$

Helmholtz equation $$\Delta g + k^2 g = \delta$$

Dirac delta function $$\delta$$

Fluid-plate boundary condition + K-H

$$\left.\frac{\partial P_m^n}{\partial z}\right)_{z=0} = \rho_\infty \omega^2 W_m^n(r) = -\frac{1}{4\pi}\int_0^R \Delta P_m^n(\rho,0)\,\frac{\partial^2}{\partial z^2}\left[\frac{e^{ikR}}{R}\right]_{z=0} 2\pi\rho\, d\rho$$

$$\rho_\infty \omega^2 W_m^n(r) = -\frac{1}{2}\int_0^R K\cdot \Delta P_m^n(\rho)\rho\, d\rho$$

Influence function K

$$K = \frac{\partial^2}{\partial z^2}\left(\frac{e^{ikR}}{R}\right)_{z=0}$$

Singular part K_s

$$K = \left(K - K_s\right) + K_s \qquad K_s = -\frac{1}{R^3} - \frac{k^2}{2R}$$

Discretization B.C.

$$\rho_\infty \omega^2 W_m^n(r_i) = \sum_{j=1}^{N} \Lambda_{ij} \Delta P_{mj}^n$$

Influence matrix Λ_{ij}

$$\Lambda_{ij} = -\frac{1}{2}\int_{r_j}^{r_{j+1}} K \cdot \rho \cdot d\rho$$

Two cases:

r_i outside $\left(r_j, r_{j+1}\right)$

K integrated numerically

r_i **inside** $\left(r_j, r_{j+1}\right)$

$K - K_s$ **integrated numerically**

K_s **integrated analitically**

Principal value

Deformation of the plate

$$w(r,\theta,t)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}q_m^n W_m^n\left(\beta_m^n r\right)\cos(m\theta)e^{-i\omega t}$$

Generalized work

$$\iint_S D\nabla^4 W_m^n W_u^v q_m^n \, dS - \rho_p h \iint_S \omega^2 W_m^n W_u^v q_m^n \, dS =$$

$$= \iint_S \Delta P_m^n W_u^v \, dS$$

Discretization

pressure jump $\left\{\Delta P_{mi}^{n}\right\} = \rho_{\infty}\omega^{2} q_{m}^{n}\left[\Lambda_{ij}\right]^{-1}\left\{W_{mj}^{n}\right\}$

generalized work

$$D\left\lfloor \nabla^{4} W_{m}^{n}\right\rfloor\left[\Delta S\right]\left\{W_{u}^{v}\right\} q_{m}^{n} - \rho_{p} h\omega^{2}\left\lfloor W_{m}^{n}\right\rfloor\left[\Delta S\right]\left\{W_{u}^{v}\right\} q_{m}^{n} =$$

$$= \rho_{\infty}\omega^{2}\left\lfloor W_{m}^{n}\right\rfloor\left[\left[\Lambda\right]^{-1}\right]^{T}\left[\Delta S\right]\left\{W_{u}^{v}\right\} q_{m}^{n}$$

$$\Big[\left[K\right]-\omega^{2}\left(\left[M\right]+\left[M_{F}\right]\right)\Big]\left\{q\right\}=0$$

Rigidity matrix

$$K_{mu}^{nv}=D\left\lfloor \nabla^{4}W_{m}^{n}\right\rfloor\left[\Delta S\right]\left\{W_{u}^{v}\right\}$$

Mass matrix

$$M_{mu}^{nv}=\rho_{p}h\left\lfloor W_{m}^{n}\right\rfloor\left[\Delta S\right]\left\{W_{u}^{v}\right\}$$

Fluid mass matrix

$$M_{Fmu}^{nv}=\rho_{\infty}\left\lfloor W_{m}^{n}\right\rfloor\Big[\left[\Lambda\right]^{-1}\Big]^{T}\left[\Delta S\right]\left\{W_{u}^{v}\right\}$$

Results

Table 1 Natural frequencies Hz for free edged circular aluminium plate (in vacuum and liquid) , radius 7.5 cm , thickness 3 mm

	Present method	Gallego-Juárez ref[11]		Amabili-Kwak ref[7]	Vacuum frequency f_{v0}^{n}
		exp	calc		
f_{l0}^{1}	566	565	527	667	1181
f_{l0}^{2}	3908	2700	2684	3336	5045
f_{l0}^{3}	10018	6533	6875	8351	11515

Table 2 Natural frequencies Hz for clamped circular aluminium plate (in vacuum and liquid) radius 7.5 cm , thickness 3 mm

	Present method	Lamb ref[4]
f_{l0}^{0}	500	500
f_{v0}^{0}	1327	1327

Table 3 Natural frequencies for free edged circular steel plate (in vacuum and liquid), radius 17.5 cm , thickness 2 mm

	Present method	Amabili-Dalp-Sant ref[9]	Amabili-Kwak ref[7]	Vacuum frequency f_{vm}^{n}
f_{l0}^{1}	65.4	67.8	78	147
f_{l0}^{2}	475	314	393	627
f_{l1}^{1}	180	159	190	328
f_{l1}^{2}	726	512	644	972

Table 4 Natural frequencies for clamped circular aluminium plate (in vacuum and liquid), radius 10 cm , thickness 3 mm

f_{v0}^{0}	f_{l0}^{0}	f_{v0}^{1}	f_{l0}^{1}	f_{v0}^{2}	f_{l0}^{2}
746.2	247.5	2905	2116	6508	5472

Table 5 Natural frequencies for free edge circular aluminium plate (in vacuum and liquid), radius 10 cm , thickness 3 mm

f_{v0}^{1}	f_{l0}^{1}	f_{v0}^{2}	f_{l0}^{2}	f_{v0}^{3}	f_{l0}^{3}
664.5	283.4	2838	2121	6477	5497

Sandwich structure - Air

Properties

$$E = 9 \cdot 10^9 \ Pa \qquad \rho_p = 139 \ \frac{kg}{m^3}$$

$$Radius \quad a = 1\,m \qquad Thickness \quad h = 1\,cm$$

Fig.1 Fluid mass coefficient vs reduce frequency for a clamped plate

$$C_{mf} = \frac{M_f}{4\pi a^3 \rho_\infty}$$

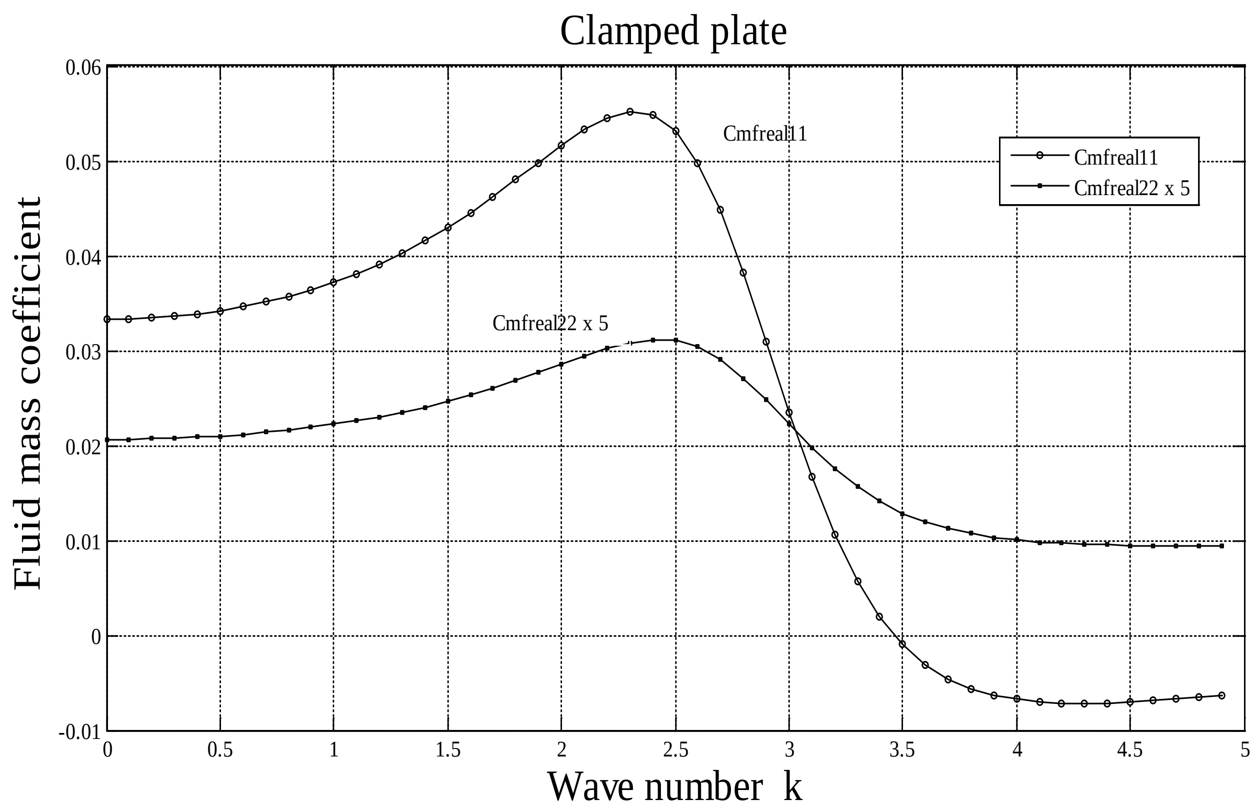

Fig.2 Fluid mass coefficient vs reduce frequency for a free edge plate

$$C_{mf} = \frac{M_f}{4\pi a^3 \rho_\infty}$$

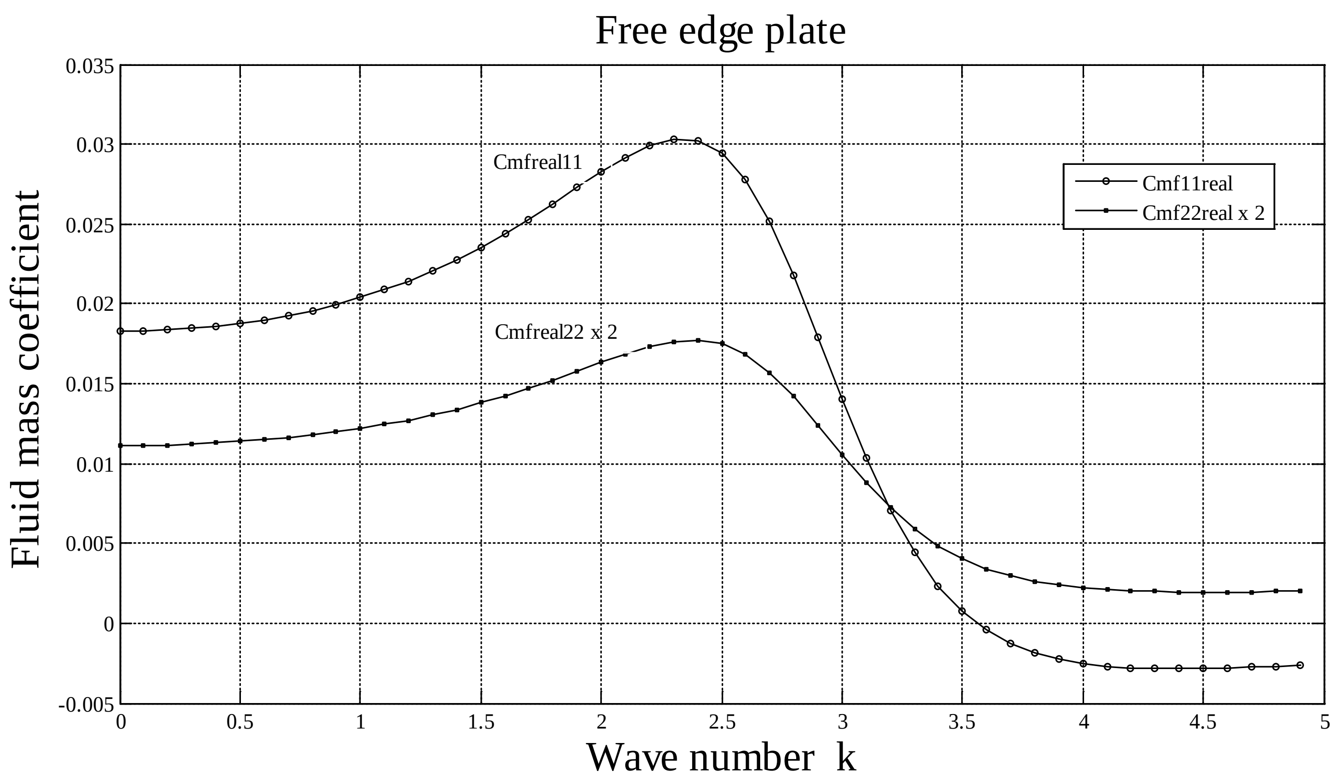

Fig.3 Fluid mass coefficient vs radius plate for a clamped plate

$$C_{mf} = \frac{M_f}{4\pi a^3 \rho_\infty}$$

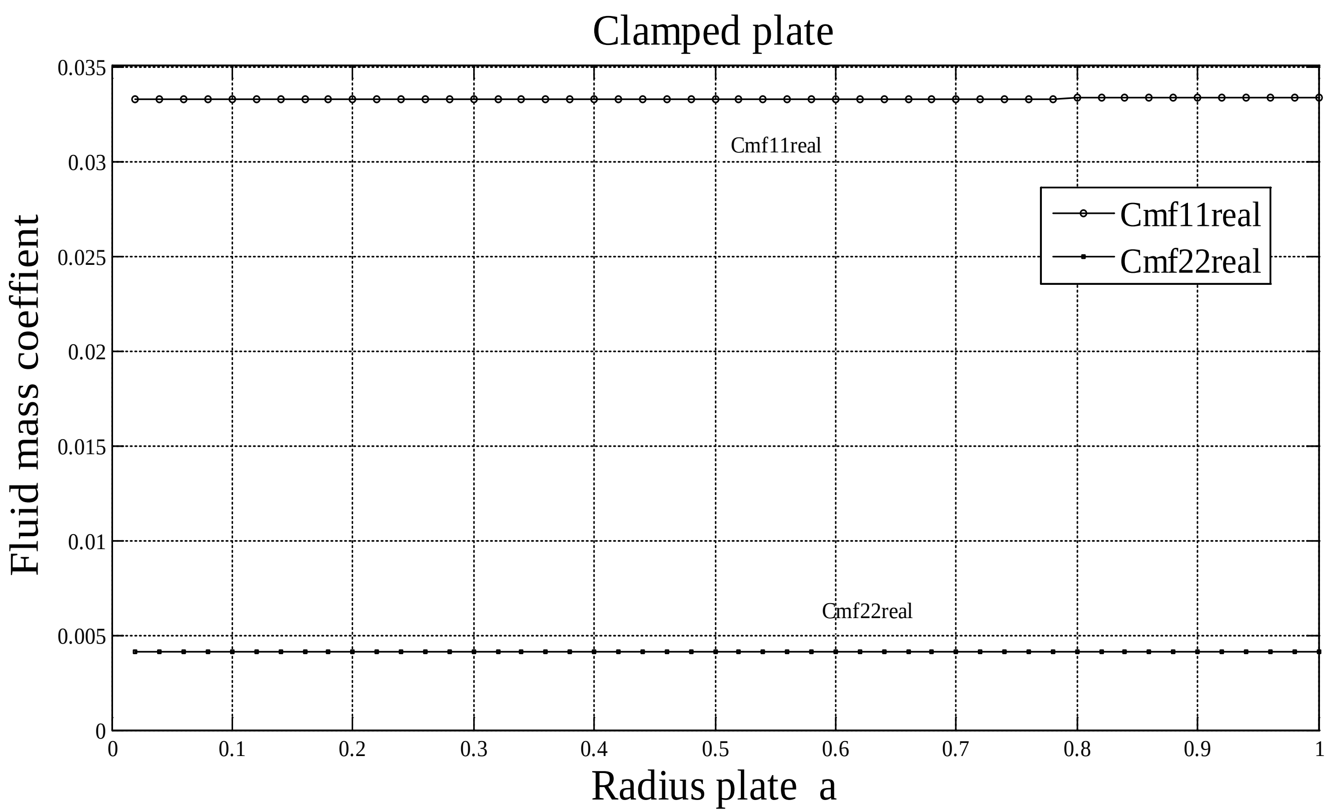

Fig.4 Fluid mass coefficient vs radius plate for a free edge p late

$$C_{mf} = \frac{M_f}{4\pi a^3 \rho_\infty}$$

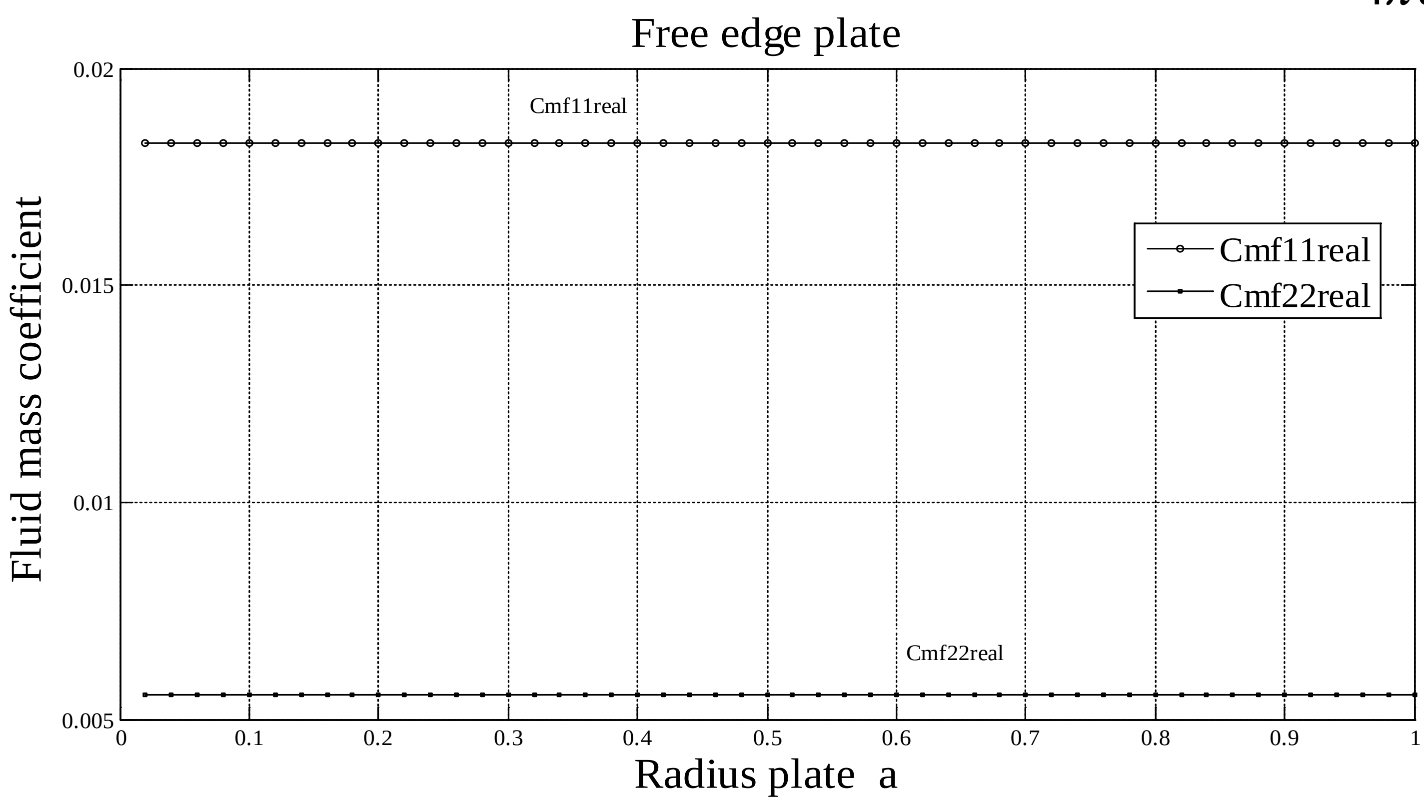

Fig.5 Frequency parameter vs radius plate for a clamped plate

$$C_{\omega} = \omega \cdot a^2 \sqrt{\frac{\rho_p}{D}}$$

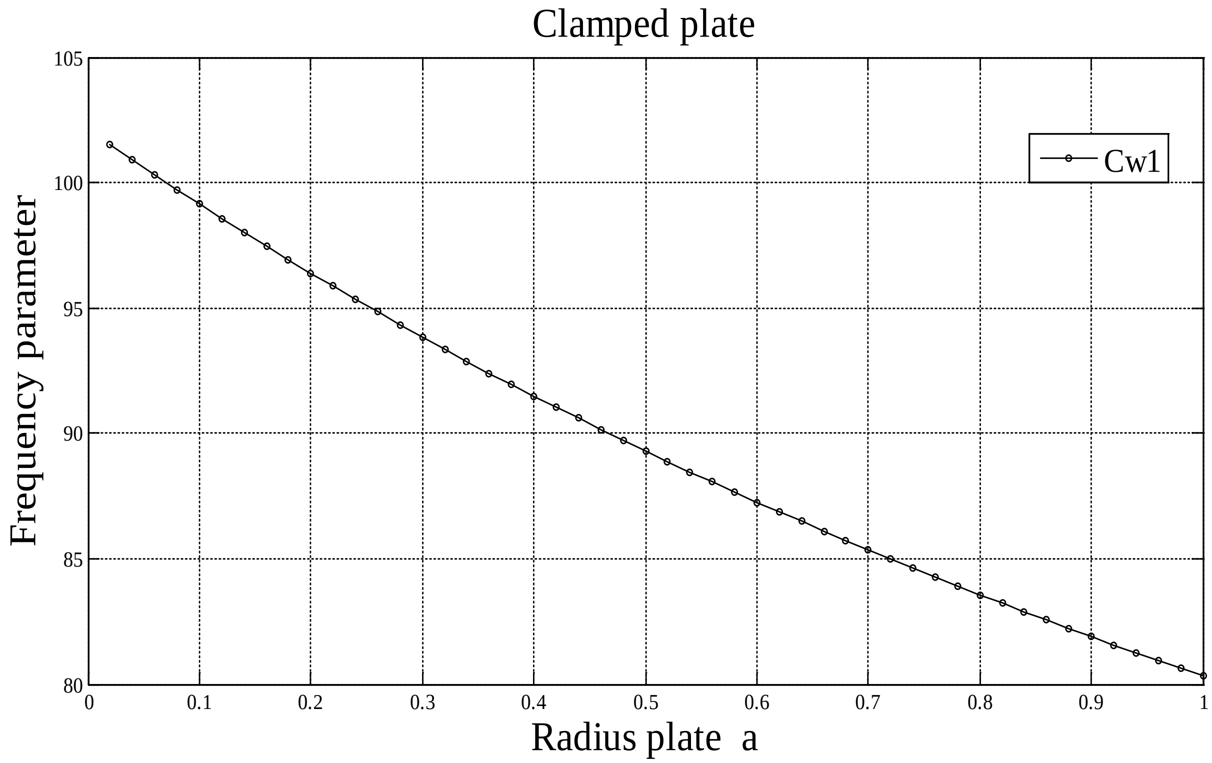

Fig.6 Frequency parameter vs radius plate for a free edge plate

$$C_{\omega} = \omega \cdot a^{2} \sqrt{\frac{\rho_{p}}{D}}$$

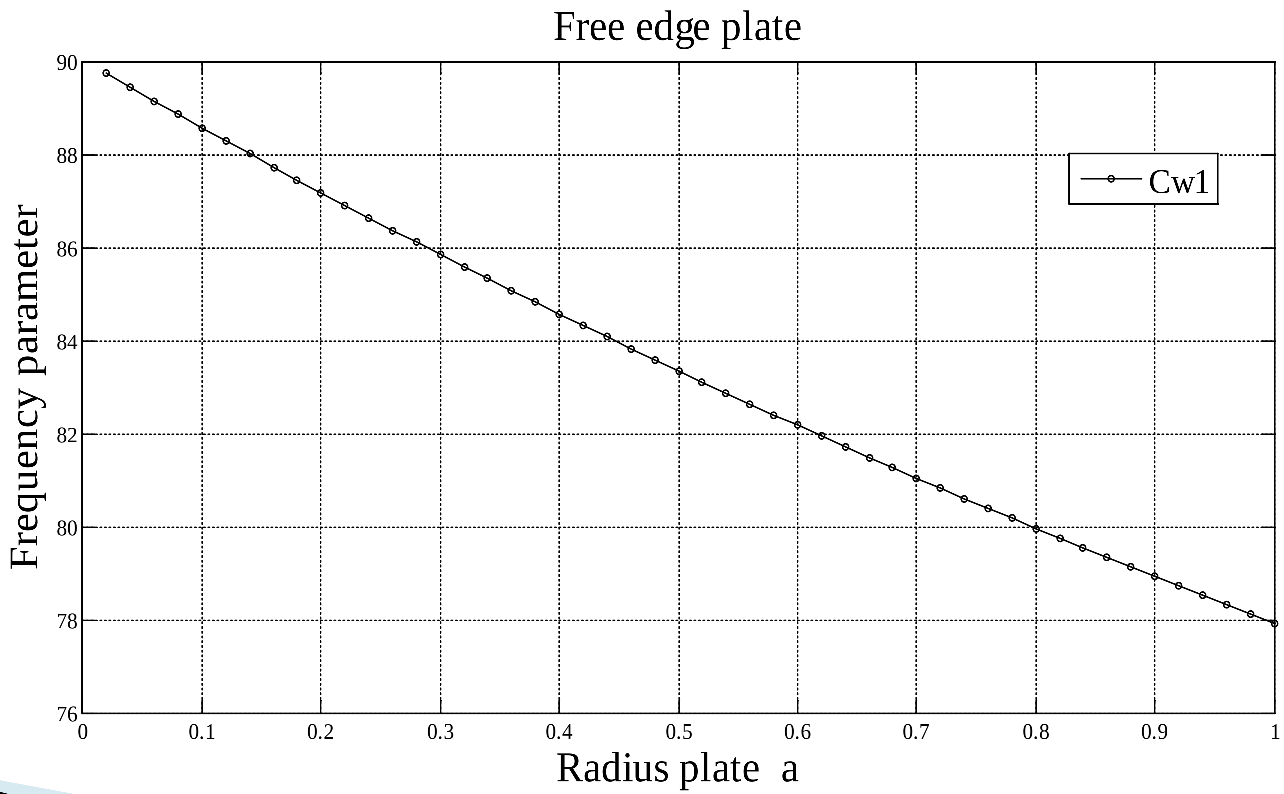

Fig.7 Frequency parameter vs radius plate for a clamped plate

$$C_\omega = \omega \cdot a^2 \sqrt{\frac{\rho_p}{D}}$$

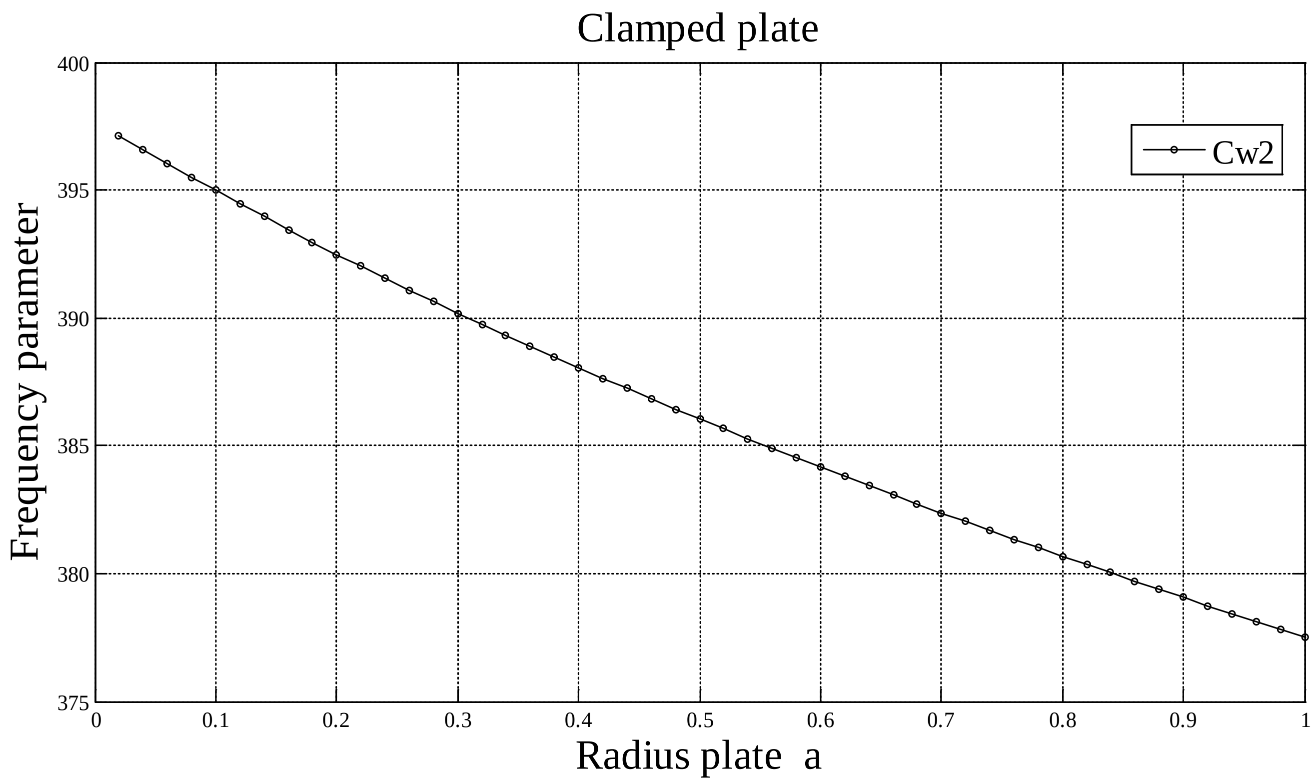

Fig.8 Frequency parameter vs radius plate for a free edge plate

$$C_\omega = \omega \cdot a^2 \sqrt{\frac{\rho_p}{D}}$$

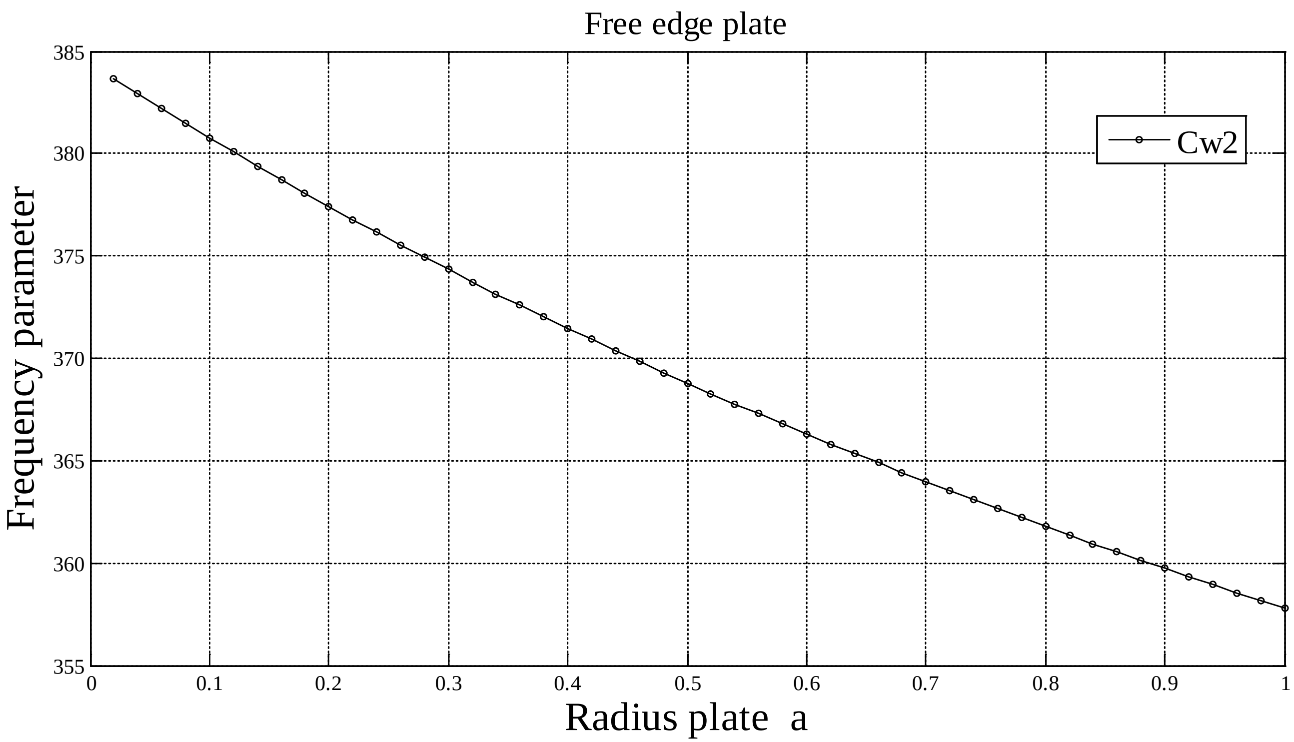

Fig.9 Frequency parameter vs relative density for a clamped plate

$$C_\omega = \omega \cdot a^2 \sqrt{\frac{\rho_p}{D}}$$

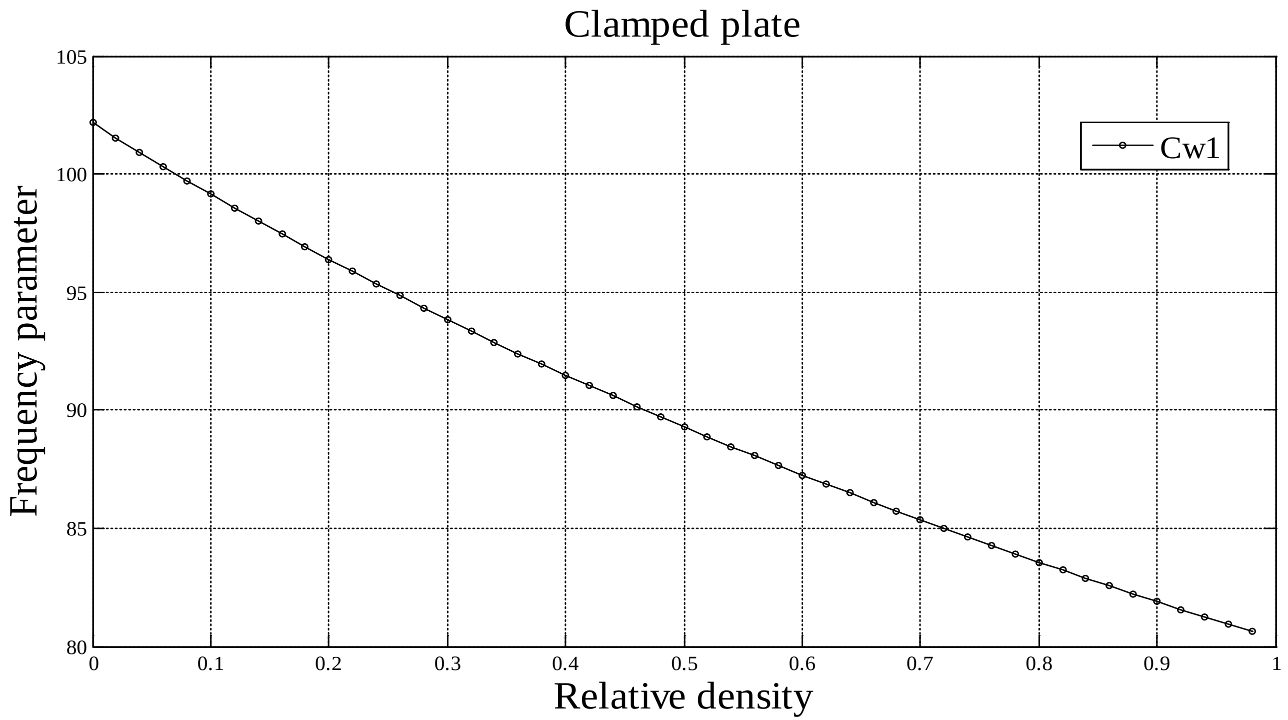

Fig.10 Frequency parameter vs relative density for a free edge plate

$$C_{\omega} = \omega \cdot a^2 \sqrt{\frac{\rho_p}{D}}$$

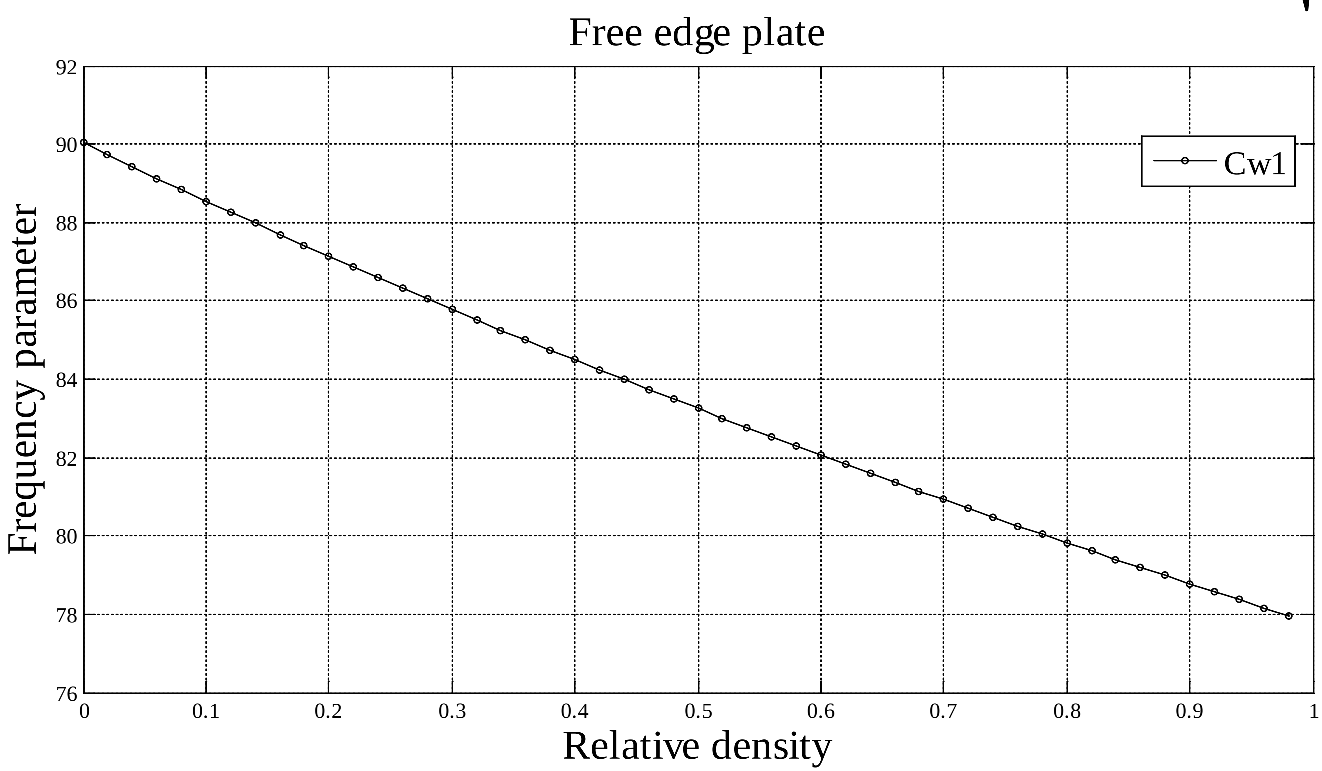

Fig.11 Frequency parameter vs relative density for a clamped plate

$$C_\omega = \omega \cdot a^2 \sqrt{\frac{\rho_p}{D}}$$

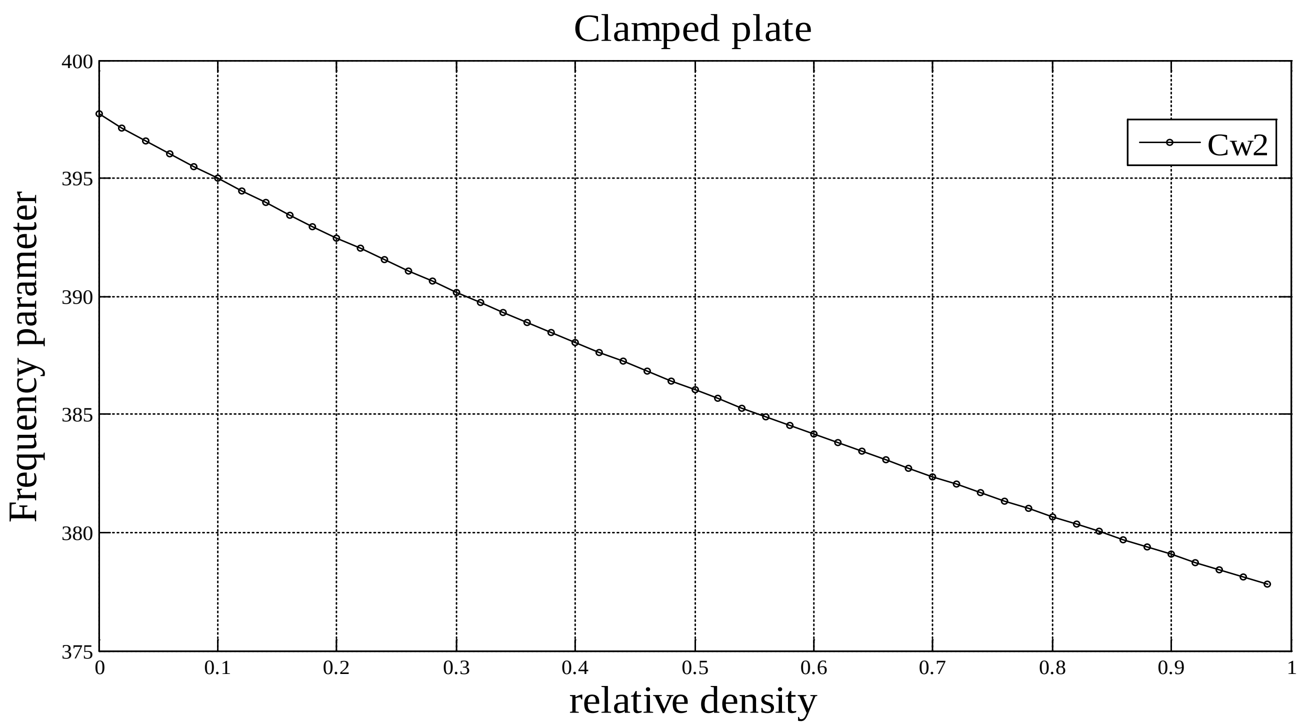

Fig.12 Frequency parameter vs relative density for a free edge plate

$$C_\omega = \omega \cdot a^2 \sqrt{\frac{\rho_p}{D}}$$

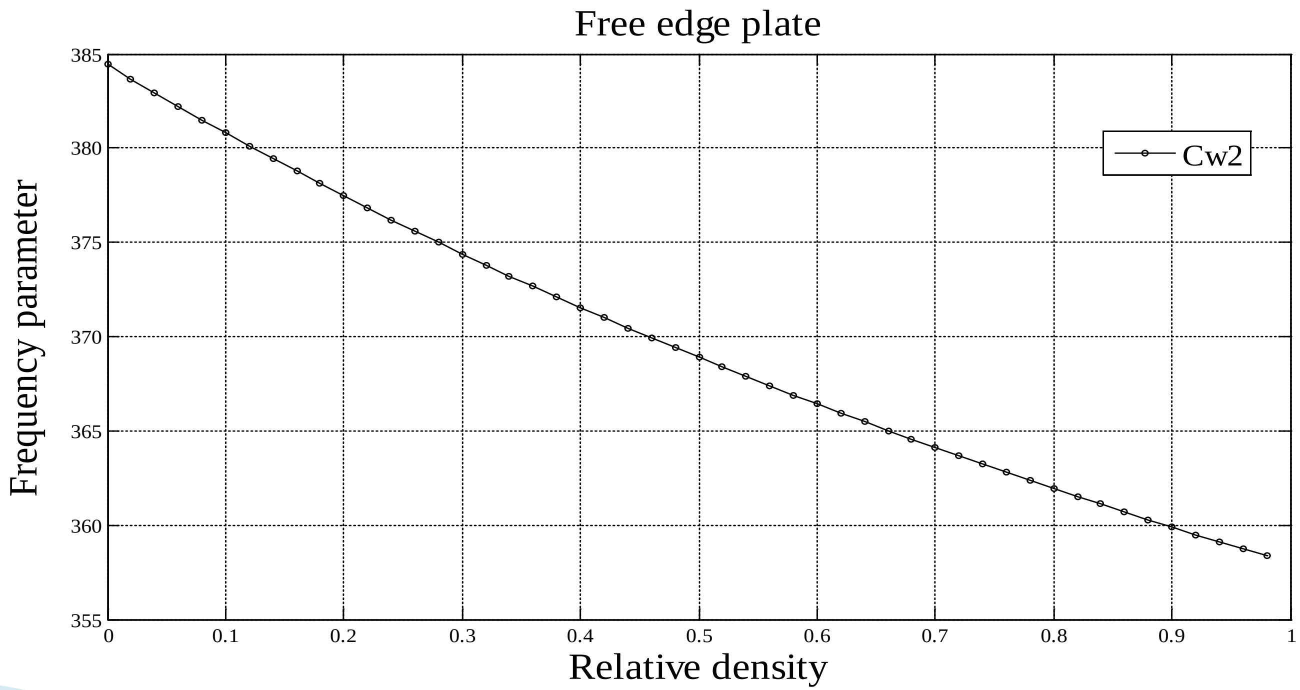

Table 6 Natural frequencies for clamped circular honeycomb plate (in vacuum and air incompressible and compresible), radius 1 m , thickness 1 cm

f_{v0}^{0}	i f_{a0}^{0} c		f_{v0}^{1}	i f_{a0}^{1} c		f_{v0}^{2}	i f_{a0}^{2} c	
39.6	31.59	31.37	154.1	146.8	144.5	345.3	333.1	339.4

Table 7 Natural frequencies for free edge circular honeycomb plate (in vacuum and air incompressible and compresible), radius 1 m , thickness 1 cm

f_{v0}^{0}	$\boldsymbol{i}$ f_{a0}^{0} $\boldsymbol{c}$		f_{v0}^{1}	$\boldsymbol{i}$ f_{a0}^{1} $\boldsymbol{c}$		f_{v0}^{2}	$\boldsymbol{i}$ f_{a0}^{2} $\boldsymbol{c}$	
35.3	30.75	30.63	150.61	140.79	136.5	343.7	332.4	337.1

Conclusions

Method calculate frequency circular plate compressible fluid

Valid any support condition
Vacuum modes
Validation incompressible methods
Light structures air important influence